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CONCERNING THE PERSIAN VERSION OF LĪLĀVĀTĪ (1587)

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(Communicated by Dr. N. R. Ray)

I

At once one of the most delightful and significant treatises in the whole history of mathematics, the *Lilāvati* of Bhāskara-cārya is a subtle exposition in which the analytical and poetical elements of thought are attractively combined. No one interested in Oriental literature can fail to fall in love with it. It forms the first part (*Paṭiganitādhyāya*) of the *Siddhānta-siromaṇi*,¹ completed by Bhāskara, of Bedar, in the Deccan around 1150 A.D., and contains, in addition to the fundamentals of arithmetic, certain problems involving simple and quadratic equations, a treatment of permutations and combinations, arithmetical and geometrical progressions, various problems in plane and solid geometry, and an exposition on the indeterminate equation of the first degree—a field in which both the Chinese and Hindus had excelled for at least half a millennium.

Since its composition in the middle of the twelfth century, *Lilāvati* has inspired a number of commentaries, translations, and editions, of which the following may be noted²:—

- (1) *Ganitākāumudī*, which has been dated 1357 A.D.
- (2) *Ganitāmpitasāgarī*, by Gangādharā, c. 1420 A.D.
- (3) The commentary of Moshadeva (prior to 1473 A.D.).
- (4) *Lakshmīdāsa Mīra*, 1501 A.D.
- (5) *Ganitāmpitakūpikā Sūryadāsa*, c. 1541 A.D.
- (6) *Buddhivilāsini*, by Ganeśa Daivajña, 1545 A.D.
- (7) *Pāṭivyākhyāna* by Vireśvarapaṇḍit.
- (8) *Lilāvati bhūṣaṇa* by Dhaneśvara, son of Vireśvara. (A later commentary than that of Sūryadāsa.)
- (9) The Persian translation made by the poet Faizi, 1587 A.D.
- (10) *Mitabhāshini*, by Ranganātha, probably c. 1630 A.D.

¹ The *Siddhānta-siromaṇi* has four parts, entitled *Paṭiganitādhyāya* (Arithmetic), *Goḷādhyāya* (Geometry and Trigonometry), *Ganitādhyāya* (Astronomy) and *Bījaganitādhyāya* (Algebra), though the division of subject-matter is not rigid.

² This list is incomplete: see also e.g. *A Descriptive Catalogue of MSS. in Mihrā' (Banerji-Sastri), III, 374-386, Patna, 1937.*

Of the full *Siddhānta-siromaṇi* we have, e.g.

- (1) The Sanskrit text, edited by Lancelot Wilkinson, Calcutta, 1842.
- (2) English translation, Lancelot Wilkinson. (Bibliotheca Indica, Calcutta, 1802).
- (3) Sanskrit text, edited by Bāpu Dova Śastri, Benares, 1866.
- (4) Commentaries on the *Siddhānta-siromaṇi* by Muneśdhar Jhā. (The Pandit, XXX-XXXIII.)
- (5) Hindi translation, Sudhākara Devedī, Benares, 1899.
- (6) Kannada translation with notes, Shri K. S. Nagarajan, 1950.

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- (11) Manorañjana, by Rāmakṛṣṇa Deva.¹
- (12) The Persian commentary of Dharma Narāyan ibn Kalyānmal Kāyath, c. 1663 A.D.
- (13) Baiswari translation by Lālchand, 1679 A.D.
- (14) Gaṇitāmṛtalahari, by Rāmakṛṣṇa, which has been dated 1688 A.D.
- (15) An elaborate commentary Sarvalbodhini, Pāṭiganitāṭikā, by Sridhara, 1717 A.D.
- (16) E. Strachey. Observations on the Mathematical Science of the Hindoos, with extracts from Persian translations of the Leelewuttee and Beej Gunnit, Calcutta, 1805.
- (17) Lilāvati (John Taylor). English translation, Bombay, 1816.
- (18) Algebra with arithmetic and mensuration from the Sanscrit of Brahmagupta and Bhāscara (H. T. Colebrooke),² London, 1817.
- (19) Faizī's translation, printed Calcutta, 1828.
- (20) Sanskrit text, printed Calcutta, 1832, 1846; Madras, 1863.
- (21) Colebrooke's translation of the Lilāvati with notes (H. C. Banerji),³ Calcutta, 1893. (Reprinted Calcutta 1927.)
- (22) Lilāvati with notes (Sudhākara Dvivedi), Benares Sanskrit Series CLIII, 1912.

A school for the study of the Siddhānta-śiromaṇi was established by Cangadeva, a grandson of Bhāskara, in the year 1206 A.D. Our present concern, however, is not with the Sanskrit tradition, but with the Persian version made by Faizī.⁴

II

There is little doubt from the foregoing that Lilāvati, and indeed the whole Siddhānta-śiromaṇi, was widely used and appraised in India.⁵ It was held in esteem in the time of Akbar, who ordered his poet laureate (Malik ush-Shu'arā), the Shaikh Abul-Faiz (Faizī or Fayyāzī) to prepare a translation in the Court language, Persian. This translation was completed A.H. 995-6 (1587 A.D.), and is mentioned in the Ā'in-i-Akbarī, which is the third volume of the Akbar Nāmah compiled by Abul-Faiz. Faizī, who was the elder brother of Abul-Faiz by some four years, was instrumental in introducing him to Akbar, whose trusted adviser he later became.

The Persian commentary based on Lilāvati and entitled Badā'i-i-funūn was written A.D. 1663-64 by Dharma Narāyan ibn Kalyānmal Kāyath and dedicated to 'Ālamgir. It would seem, therefore, that during the period of the Mughal emperors the mathematical work of Bhāskara received general recognition in learned circles.

We have noted MS. copies of Faizī's version in the British Museum (MS. Add. 5649, dated A.D. 1777),⁶ the Library of the India Office (MSS. 1998 dated A.D. 1606 at Shāhjahānābād, 1999 dated A.D. 1777, and 2000

¹ Also of uncertain date. MS. 2816 of Cat. Sanskrit MSS. India Office, Part V, p. 1008 (Ezzelino), is in Devanāgarī script, c. 1750.

² Donated in Section III *infra* 34 C.

³ Donated in Section III *infra* 34 B.

⁴ بنی

⁵ e.g. 'Atā-ullāh Rashīdī bin Ahmad Nādir produced a Persian version of Bijaganita in A.D. 1634-5 for Shāh Jahān.

⁶ Cat. of the Persian MSS. in the British Museum (C. Rien), II, p. 149 (1881).

dated A.D. 1779),¹ and the John Rylands Library in Manchester (MS. 699c. A.D. 1729).² There is also a copy listed by Pertsch,³ and no doubt others exist. We have translated the Manchester MS., which is not a complete version, and it will now be briefly discussed.

III

This Persian MS. listed by Michael Kerney comprises ff. 115v. to 134r., the script being enclosed by a thick border, with wide marginal spaces in which additional writing is generally to be found. It is incomplete in the sense that it is a selection of examples taken from Lilāvati to illustrate the methods of commercial arithmetic. The procedure adopted is that of stating a general rule of calculation, followed by an appropriate numerical example.

The first example cited is from Section IV of Chapter IV of Lilāvati,⁴ dealing with 'Investigation of Mixture' and is as follows:—'A certain man possesses eight rubies and another possesses ten emeralds, a third a hundred pearls and a fourth five diamonds.....'

'Each of them made a friendly gift, to each one of the others, of a single article from his own stock. The owner of rubies gave one ruby to each of his three friends. Similarly the owners of emeralds, pearls, and diamonds, each gave one of their articles to each of the other three friends.' As a result of the transaction 'nobody owed anything to anyone else' and each possessed a total stock of the same value.⁵

Problem: to find the price of each kind of gem. 'The mathematical statement may be written thus:

5	100	10	8	[Numbers of gems].
1	1	1	1	[Number in gift].

'As the transaction was made by each person giving away one article, we multiplied one by four, and obtained four; next we subtracted four from eight and obtained four as remainder. We then again subtracted four from ten and obtained six. Once more four was subtracted from one hundred, and thus ninety-six remained. By subtracting four from five only one remained.⁶

4	6	96	1
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cf. Shary II/1, p. 16, no. 301
Muzavi I, p. 142a,
no. 1234-36

¹ Cat. of the Persian MSS. in the Library of the India Office (H. Ethé), I, pp. 1112-1113 (1903).

² Bibliotheca Lindesiana: Handlist of Arabic, Persian and Turkish MSS. (Michael Kerney), p. 136. Privately printed, 1898.

³ W. Pertsch. Berlin Cat., p. 1031.

⁴ C. p. 45. B, p. 56.

⁵ If the relative values of rubies, emeralds, pearls and diamonds are represented by a, b, c, d , then after the gifts have been made $5a+b+c+d = 7b+a+c+d = 9c+a+b+d = 2d+a+b+c$.

⁶ This leads to

$$4a = 6b = 96c = d (= k \text{ say}).$$

'Furthermore, we assumed a number, viz. ninety-six, and divided it by each one of the remainders. The quotient obtained with four (as divisor) is twenty-four, with six is sixteen, with ninety-six is one, and with one is ninety-six. Hence the price of a ruby is twenty-four and the price of an emerald is sixteen, the price of a pearl is one, and the price of a diamond is ninety-six. Hence each man possessed articles of price 233 *nishkas*.¹ 'Another solution is this:—we multiplied all the remainders together, so that when 4 was multiplied by 6 it gave 24, and when 24 was multiplied by 96 it gave 2304, and when this was multiplied by unity it remained the same. Then we divided the whole product by each remainder. The quotient in the first case was 576, and in the second 384, in the third 24, and similarly in the fourth 2304. According to the calculation each of them possessed articles worth 5592 *drammas*,² and when we convert these *drammas* into *nishkas* there are 233 *nishkas*, as has been mentioned in the first method.'

The second discussion concerns the Rule of Three, and is taken from *Lilāvati*, Chap. III, Sec. VI.³ 'The given thing of which the quantity is to be found is named *pahal*,⁴ secondly the price is named *parmān*,⁵ and thirdly the value of the thing by which we estimate the amount of it is called *achhā*⁶

'Example. If two and a half *palas*⁷ of saffron is bought for three-sevenths of a *nishka*, what can be bought for nine *nishkas*? I write *parmān* 3/7, *pahal* 5/2, and *achhā* 9/1, in this way:—

3	5	9
7	2	1

We multiplied *pahal*, which is 5/2, by *achhā*, which is 9, and obtained $\frac{45}{2}$ as product. We then divided it by 3/7 which is *parmān*, and the quotient is 52 *palas* and 2 *karshas*⁸ of saffron for 9 *nishkas*.⁹

The text then proceeds to the Rule of Three, inverse proportion. 'As an instance, if a girl of sixteen years can be bought for 32 *ashrajis*¹⁰ (gold coins), how many *ashrajis* will be required to buy a girl of twenty years? Also an ox which has been worked for two years can be bought for 4 *nishkas*. What will be the cost of an ox which has been worked for six years?

¹ If we put $k = 96$ (the l.c.m. of 1, 4, 6, 96), then $a = 24$, $b = 16$, $c = 1$, $d = 96$; whence the total

$$5a + b + c + d \text{ (say) is } 233.$$

MS. نَشِك . B has *nishkas*, C. has *nishkas*.

² If we make $k = 4 \times 6 \times 96$, then $a = 576$, $b = 384$, $c = 24$, $d = 2304$. The total stock of each man = $5a + b + c + d$ (say) = 5592.

MS. درم

³ C, p. 33. B, p. 39.

⁴ MS. پَہال

⁵ MS. پَرَمَان

⁶ MS. اَچھا

⁷ MS. پال . B has *palas*.

⁸ MS. کَرشہ . C has *karshas*. B has *karshas*.

⁹ MS. اَشْرَجِي . Cf. C, p. 34. C, p. 41. The Sanskrit has *nishkas*.

'I first write *parmān* which is 16, then *pahal* which is 32, and finally *achhā* which is 20, in this way¹:—

16	32	20
1	1	1

We multiplied 32 by 16, which gave 512. We then divided it by 20, and obtained $25\frac{1}{2}$ as quotient.² So the girl of twenty years is valued at $25\frac{1}{2}$ *ashrajis*.³

'Now I discuss the problem of the ox. First I write *parmān* which is 2, next *pahal* which is 4, and finally *achhā* which is 6, in this way:—

2	4	6
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We multiplied 4 by 2, thus obtaining 8. By dividing 8 by 6, we obtained $1\frac{1}{3}$. In this way they bought the ox which had been worked for six years for $1\frac{1}{3}$ *nishkas*.³

There follows now a discussion on compound proportion, in which five, seven, nine, or eleven terms are involved. We reproduce the first two of four examples which are given in illustration of the Rule of Five:—

'For instance, I wish to know, that if on 100 rupees there is an interest of 5 rupees in one month, what will be the interest on 16 rupees in twelve months? Also, if under the above conditions we obtain 9½ rupees interest on 16 rupees (as capital), what is the time?

'Firstly, I write down *parmān* which is one month, 100 rupees (capital), and 5 rupees as interest. Next I write down *achhā* which is 12 months and 16 rupees, and below that a zero,⁴ thus:—

1	12
100	16
5	

¹ The terms *parmān*, *pahal*, and *achhā* now conform to inverse proportion.

² MS. ۲۵

³ MS. ۲۵

⁴ MS. ۰

⁵ This problem is followed by a similar one relating to the 'touch and weight of gold'. See C, p. 34. B, p. 41. A further example is also given which relates to the vessel used in the measurement of *shawls*.

⁶ MS. صفر. In the MS. a dot is used to represent the unknown quantity which it is required to evaluate.

amāna
echa

19

1E:

We brought the 5 under the 16, and took the zero under the 100, in this way:—

1	12
100	16
	5

After that we multiplied 12 by 16, which became 192. Then we multiplied 192 by 5, and obtained 960. This is *achhā*. Then we multiplied 1 by 100, giving 100. This became *parmān*. As *achhā* was greater than *parmān*, I divided it into *parmān*. The quotient is $9\frac{3}{5}$ rupees.¹ So the interest on 16 rupees for 12 months, according to the above conditions, is equal to that sum.....²

The other example is like this: the time is not known but the capital and interest are given. First of all we wrote down *parmān*, which is 100 rupees (capital) and 5 rupees as interest for one month, then *achhā*, which is 16 rupees (capital) and $\frac{48}{5}$ rupees as interest, thus:—

1	
100	16
5	48/5

After that we put the sum of $\frac{48}{5}$ below the 100; and the 5 which is now below the 100, we brought below the 16, like this:—

1	
100	16
48/5	5

¹ MS. १

² Let x = interest required.
Then by the principle of proportion—

$$100 : 16 \times 12 = 5 : x, \text{ giving } x = 9\frac{3}{5}.$$

Next we multiplied 100 by 48, which became 4800, below which is 5 (i.e. $\frac{4800}{5}$). Then we divided the new fraction by 80. Then quotient was 12. Thus we found that the number of months is 12.³

The example used to illustrate the Rule of Seven is:—

There is a sheet of high-quality silk which is of length 8 *cubits*² and width 3 *cubits*; and eight such sheets can be bought for 100 nishkas. What then will be the price of another sheet of the same quality which is $3\frac{1}{2}$ *cubits* long, $\frac{1}{2}$ *cubit* wide?

I write the *achhā* like this:—

3/1	7/2
8/1	1/2
8/1	1/1
100/1	

The solution is then obtained, as in the Rule of Five, by the method of proportion,³ and is Nishka 0, drammas 14, panas 9, kākint 1, cowry shells $6\frac{3}{4}$.⁴

It is not proposed to indicate here the development of this method in the Rule of Nine and the Rule of Eleven; in the case of nine factors the text follows the Sanskrit original in giving the example involving the cost and dimensions⁵ of wooden planks, and in the case of eleven factors the same example is repeated but with two groups of coolies, differing in number, to transport them.⁶

The next discussion deals with barter, and the following example is given:—

If 300 mangoes can be bought for 16 panas, and 30 pomegranates for 1 pana, find how many pomegranates of this kind can be obtained for 10 of these mangoes.

¹ C, p. 37. B, p. 44.

² MS. دست

³ Let x denote the required price. Then, using the areas of silk, we have

$$8 \times 3 \times 8 : 1 \times \frac{1}{2} \times \frac{1}{2} :: 100 : x$$

$$\text{whence } x = \frac{1 \times 7 \times 1 \times 100}{8 \times 3 \times 8 \times 2 \times 2} \text{ nishkas.}$$

⁴ Answer obtained using 4 kākintā (MS. کاکنی) = 1 pana

$$\begin{array}{ll} 16 \text{ panas} & \dots 1 \text{ dramma} \\ 16 \text{ drammas} & \dots 1 \text{ nishka.} \end{array}$$

(MS. has برانک in place of cowry shells.)

MS. Folio 118 gives tables of quantities which correspond with those of Līlāvati, Chap. I.

⁵ C, p. 37. B, p. 45. The 'finger' measurement, انگشت occurs.

⁶ In Sanskrit the distance in leagues over which the planks have to be carried is different in this case.

'I write the *parmān* and *achhā* thus:—

16	1
300	30
10	

'Then, the amount which is in the middle of *parmān*, i.e. 300, and the amount which is in the middle of *achhā*, i.e. 30, we interchanged in position, thus:—

16	1
30	300
10	

'Then, in accordance with the foregoing method, I find that 16 pomegranates are equivalent to 10 mangoes.¹

The next illustration is taken from *Lilāvati*, Chap. IV,² and deals with Investigation of Mixtures, being a problem on capital and interest in which both these values have to be found.³ 'For example, on 100 rupees it has been decided to pay 5 rupees per month (as interest). After one year 1000 rupees were paid (altogether).⁴ If I wish to know the capital and the accrued interest, I write in this way:—

1	12
100	1000
5	

¹ There are two instances of proportion, as noted by Sūryadāsa, in this problem—

(i) To find the number of pomegranates x obtainable for 16 panas.

$$30 : x :: 1 : 16 \\ \text{giving } x = 30 \times 16.$$

Thus 30×16 pomegranates are equivalent to 300 mangoes.

(ii) To find the number of pomegranates y required.

$$300 : 10 :: 30 \times 16 : y \\ \text{giving } y = \frac{30 \times 16 \times 10}{300} = 16.$$

² C, p. 39. B, p. 49.

⁴ Meaning capital plus accrued interest.

(g) To fill any omes in the society on being duly elected thereto.

'We multiplied 100, which is *parmān*, by its time, which is one month; it became 100. After that we multiplied 5, which is *pahal*, by 12, which is the total time interval; it became 60. Then we added 100 to 60 and put down the total separately. Next we multiplied 60 and 100 in turn by 1000. The product of 60 and 100 is 60,000,¹ and the product of 100 and 1000 is 100,000. We then divided 100,000 by 160; the quotient was 625. This is the amount of the principal. If we divide 60,000 by 160 the quotient will be 375. This is the accrued interest. If we add 625 and 375 it will give 1000.²

The same problem is then solved by the method of supposition, rather like the solution of a simple equation. 'We imagine a number, and every step which was taken in the case of the Rule of Five, we took with this supposed number. . . . ' This number was taken as unity.

1	12
100	1
5	

'In this way, as with the Rule of Five, we brought 5, which is under *parmān*, below 1, which is *achhā*; and brought 0, which is below 1, underneath 100. All the numbers of *achhā* and *parmān* should then be multiplied among themselves separately. The product of *parmān*, which is 100×1 , is 100. The product of *achhā*, which is 12×5 , is 60. We then divided 60 by 100; the quotient is $\frac{3}{5}$. Next we added the supposed number, which is 1, to it. The total came to $1\frac{3}{5} = 8/5$. Next we multiplied 1000, which is given, with unity, which is the supposed number. It became 1000. Then we divided 1000 by $8/5$, which became 625, which is the interest.'

Two further examples follow which involve (i) a sum of money lent in three portions at different rates of interest for different periods, and (ii) the sharing of an aggregate sum of money earned jointly by three partners who each supplied a different initial capital.³

The next two problems concern purchase and sale in the bazaar; it will suffice to quote one of them, as they follow closely *Lilāvati*, Chap. IV, Section III:—

'As another instance, one can obtain one *pala* of camphor for two *nishkas*, and one *pala* of sandal for $\frac{1}{2}$ *dramma*⁴ and $\frac{1}{2}$ *pala* of aloe-wood for $\frac{1}{2}$ *dramma*. A man gave a *nishka*, and asked for one part of camphor, 16 parts of sandal, and 8 parts of aloe.' The answers in price and

¹ MSS. ۶

² By the modern method, if the principal = P , interest = I , total amount = A , rate per cent per month = r for a time in years = T , then,

$$I = \frac{P \times r \times 12 \times T}{100}$$

$$\therefore A = P + I = P \left[1 + \frac{r \times 12 \times T}{100} \right].$$

$$\text{giving } P = \frac{A \times 100}{100 + (r \times 12 \times T)} \text{ and } I = \frac{A \times r \times 12 \times T}{100 + (r \times 12 \times T)}$$

³ C, pp. 40-41. B, pp. 51-53.

⁴ MS. درم

quantity for camphor, sandal and aloe respectively are $14 \frac{2}{9}$ (dramma), $\frac{4}{9}$ (pala); $\frac{8}{9}$, $\frac{64}{9}$; $\frac{8}{9}$, $\frac{32}{9}$.

Finally, a question is taken from Lilāvati, Chap. IV, Section II, on fractions, and relates to the familiar exercise about the cistern:—'There is a tank into which water flows from four different directions. One of its inlets is such that if we leave it to itself, it will fill the tank in one day. The second one fills it in $\frac{1}{2}$ day, the third one in $\frac{1}{3}$ day, and the fourth one in $\frac{1}{6}$ day. If we open all four of them together, in what portion of a day would they fill it?'

It is written in this way¹:—

1	1	1	1
1	2	3	6

We divided every number written below by the number written above. According to the previously-mentioned method of the division of fractions we obtained then

1	2	3	6
1	1	1	1

After this we added the numbers written above, which are quotients, thus obtaining 12. Next we divided unity by 12. Thus it is $\frac{1}{12}$ th part of a day, which is a half nichkeri,² in which it is filled.

We conclude in the name of God, virtue, and victory.'

Such are the main features of this Manchester MS.; dealing only with a part of Lilāvati, and that part which concerns primarily business transactions, it is in the tradition of commercial arithmetic first established by the mediaeval Sanskrit treatises.

IV

This paper, in its small way, calls further attention to the Hindu genius for mathematics, to its unique contribution to mathematical thought and symbolism which has survived in virtue of its universality. It became known to mediaeval Islām, and thence to Latin Christendom, long before, Bhāskara, and was studied with a rare sympathy and enthusiasm by Al-Birūnī; whilst we find it again in the Persian works in the days of the Mughal emperors, when Europeans in India were, had they appreciated it, indeed at the fountain-head. Not, however, until the early nineteenth century did the editions of E. Strachey, J. Taylor, and H. T. Colebrooke see the light of day.

We conclude by expressing our appreciation of the generous assistance afforded by the staff of the John Rylands Library, particularly the kindly co-operation of the late Dr. Henry Guppy. Also we are indebted to Roshan Zamir of the Education Dept., Tabriz, Azerbaijan, who kindly helped in checking the translation.

¹ The procedure amounts to this:—If the inlets separately can fill the tank in 1, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ day, then they also separately fill 1, 2, 3, and 6 parts of the tank in one day, and thus collectively fill 12 parts of the tank in one day, i.e. collectively they fill the tank in $\frac{1}{12}$ day.

² MS. نیچری

CONSTRUCTION OF FERTILITY TABLES FOR INDIA AND STUDIES THEREFROM

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The provisional figure of total population in India has been given as 356.89 millions* on 1st March, 1951. This records an increase of 13.4% over 1941 figures. The rate of increase in this intercensal period also is rather high as in previous one. Fear of over-population is therefore haunting the mind of every intelligent citizen. Many social and economic difficulties now being experienced have been attributed to this rapid growth of population.

Growth of population depends mainly on the age and sex composition of the population and the fertility rate prevailing in the country. In India among 356.89 millions of population there are now 173.51 millions of females. Sex-ratio thus works out at 946 females for every 1,000 males in 1951 as against 940 in 1941. Proportion of females thus shows an increase on the whole but its influence on growth of population cannot be ascertained unless the age-distribution of this increased female population is known because such increase must be in child-bearing ages in order to be effective. This age-distribution will become known when the census report will be published. Even then it would not be possible to measure accurately the future contribution of this increase to population unless the rate of fertility among women is known for all ages of child-bearing period. But the census authorities, for reasons best known to them, have so far refrained from collecting information in a form suitable for fertility studies. It is therefore apparent that diagnosis of the real cause of rapid growth of population in India may not be feasible from census figures.

In the absence of such complete enumerations, we must have recourse to sample surveys. But sample studies also have been very rare in this direction in India. Only one fertility table has been published so far but that also from a few hundreds of families in Cochin, a very small State in India. As the conditions in Cochin differ greatly from those obtaining in other parts in India and as the size of the sample was very small, these rates could not be used to represent the fertility of this vast subcontinent. For fertility studies in India, we have always been using the fertility tables of foreign countries like Ukraine or Japan. But the use of such tables in conjunction with our female population is not likely to yield satisfactory results because the fertility curve of India has a different shape from that of the above foreign lands. The population studies in India have always faced this difficulty and the necessity of a fertility table derived from our own population has long been recognized but no all-out attempt has as yet been made.

The main object of this paper has therefore been to derive specific fertility rates for different provinces of India, to study the influence of age-difference and financial position of the couple on these rates, to construct a combined fertility table and to study population trends.

* Exclusive of Jammu and Kashmir State and certain tribal areas.